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Scaling of flow quantiles and mean catchment fluxes and storage provides empirical formulation of the flow duration curve

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Introduction

- The flow duration curve (FDC) provides a simple representation of the magnitude and frequency of catchment runoff response, widely useful tool for both investigation of runoff generation processes and river management.
- Despite the wide array of methods available for prediction in ungauged basins, there remains a need for reliable methods of FDC prediction that can be universally applied.
- Recent studies[1,2,3] point to the control of quickflow and baseflow on the shape of the flow duration curve.
- We explore the intrinsic scaling between flow quantiles and mean annual values of streamflow $\overline{Q}$, baseflow $\overline{Q_b}$, quickflow $\overline{Q_d}$, and deep storage $\overline{S_{deep}}$.
- We demonstrate that these scaling relationships can be leveraged to reconstruct the FDC from mean annual fluxes using regression analysis.

Catchment selection

The study catchments are located in the conterminous United States and are a subset of the MOPEX[4] dataset:

- 360 perennial catchments
- 25 years continuous daily streamflow

We tested the validity of the derived regressions on 428 catchments with 25 years of data from the CAMELS[5] data-set.

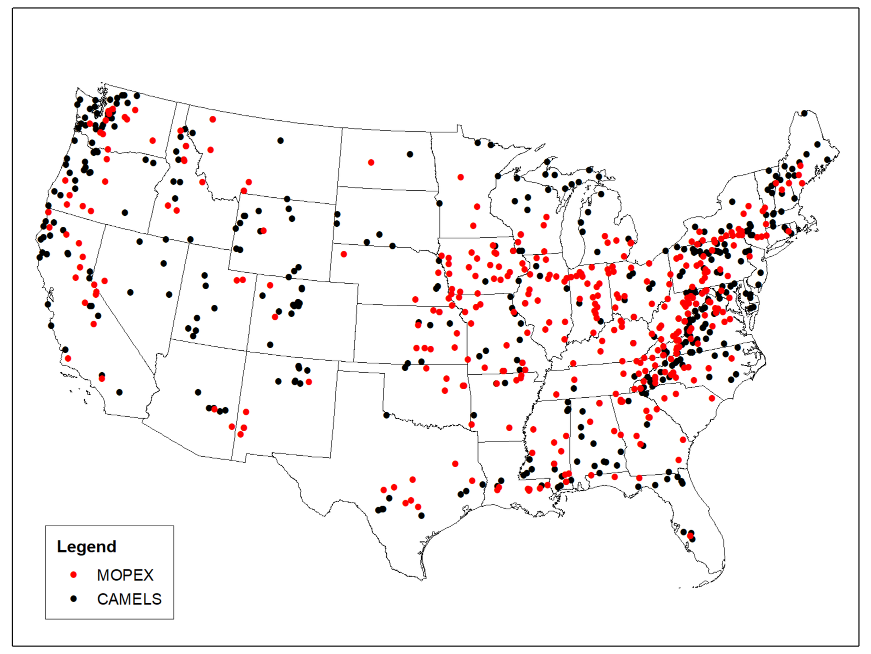


Figure 1. Locations of catchments used in this study.

Methods

- Baseflow (Q_b) and quick flow (Q_d) determined using a standard digital filtering method[6]:

$$Q_{b,i} = \epsilon Q_{b,i-1} + \frac{1-\epsilon}{2}\left[Q_i + Q_{i+1}\right] \qquad Q_b(t) < Q(t)$$

- Storage (S) was calculated using recession behavior and a linear reservoir model of storage:

$$\frac{dS_{deep}}{dt} = -Q_b$$

$$S_{deep} = k_b Q_b$$

$$(Q_{b,i-1} - Q_{b,i}) = \frac{1}{k_b}\left(\frac{Q_{b,i-1} + Q_{b,i}}{2}\right)$$

- Correlations were calculated with Pearson's r:

$$r = \frac{cov(X,Y)}{\sigma_X \sigma_Y}$$

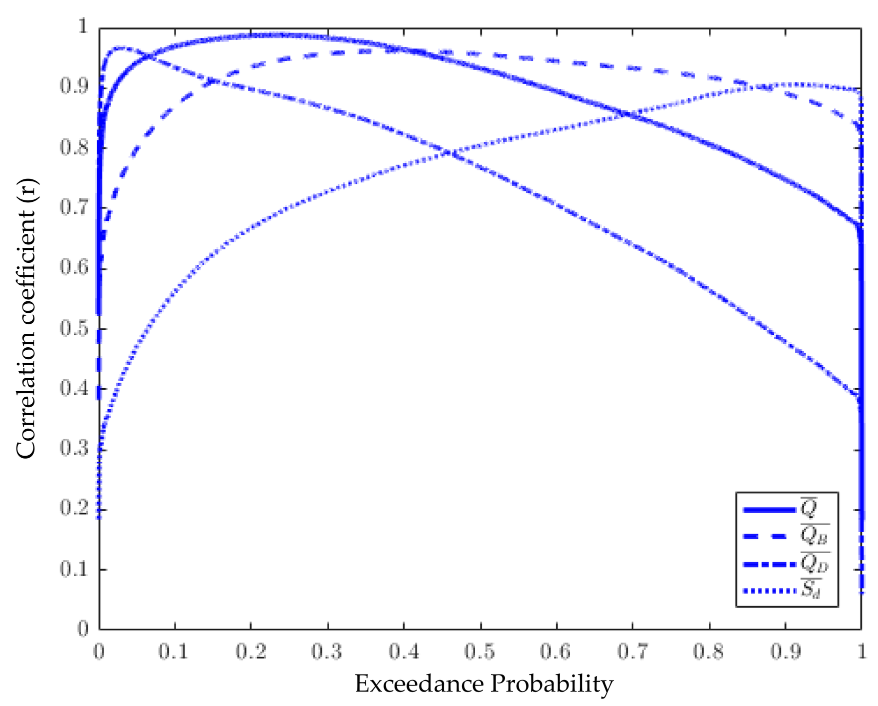


Figure 2. Scaling of several flow quantiles (x-axes) with mean daily discharge, direct flow, baseflow, and deep storage (y-axes) for all catchments. Pearson's correlation coefficient r is given for each scaling relation

Results: Regression Models

Linear regression was used to determine functional relationships between mean annual fluxes and flow quantiles and to determine predictive equations of FDCs:

Model	Equation
Q	$ln(Q_x) = x_1 ln(Q) + C$
QS_d	$ln(Q_x) = x_1 ln(Q) + x_2 ln(S_{deep}) + C$
DB	$ln(Q_x) = x_1 ln(Q_b) + x_2 ln(Q_d) + C$
DBS_d	$ln(Q_x) = x_1 ln(Q_b) + x_2 ln(Q_d) + x_3 ln(S_{deep}) + C$

Table 1. Regression models assessed in this analysis.

The accuracy of the predictive equations is analyzed using Nash-Sutcliffe model efficiency statistics:

$$NSE_{between}(i) = 1 - \frac{\sum_{j=1}^{n}(Q(i)_{pred,j} - Q(i)_{obs,j})^2}{\sum_{j=1}^{n}(Q(i)_{obs,j} - \overline{Q(i)_{obs}})^2}$$

$$NSE_{within} = 1 - \frac{\sum_{i=1}^{99}(Q_{pred,i} - Q_{obs,i})^2}{\sum_{i=1}^{99}(Q_{obs,i} - \overline{Q_{obs}})^2}$$

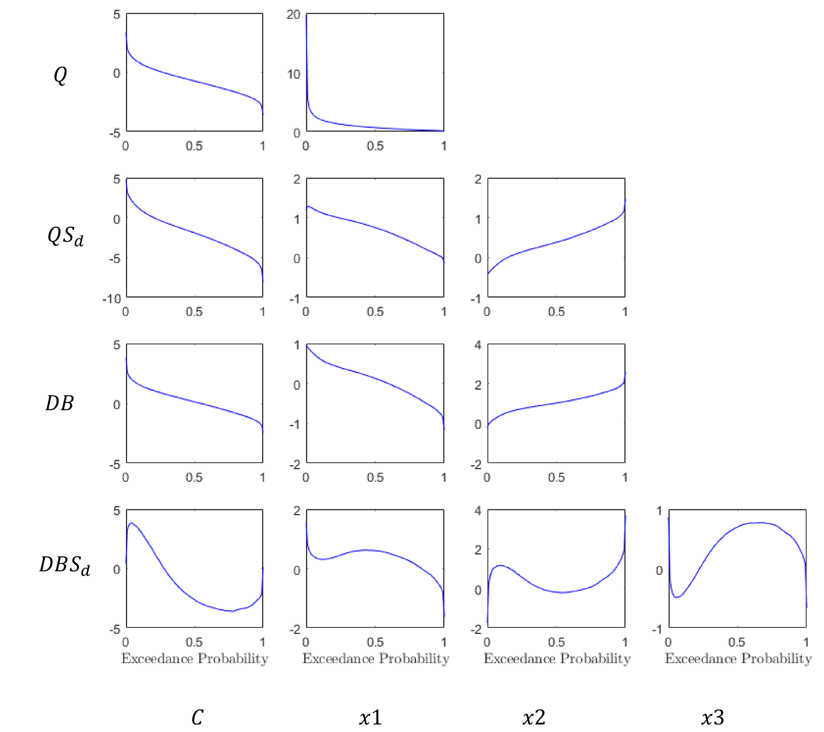


Figure 4. Parameters (y-axes) of linear regression models for flow quantiles (x-axes)

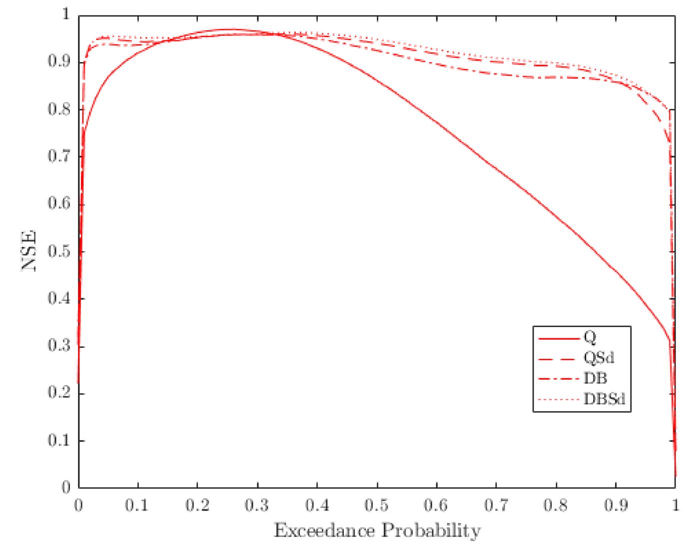


Figure 5. Between-catchment NSE values per flow quantile

Table 2. Percent of catchments with NSE values above specified levels for the calibration data-set (MOPEX)

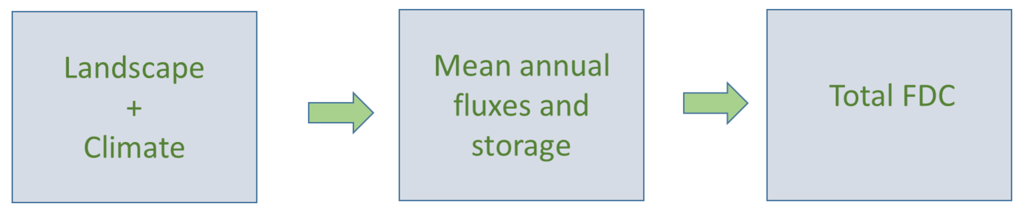
NSE	Q	QS_d	DB	DBS_d
> 0.5	95	99	99	99
> 0.6	94	99	99	99
> 0.7	91	99	99	98
> 0.8	84	98	98	98
> 0.9	75	92	91	91

Table 3. Percent of catchments with NSE values above specified levels for the validation data-set (CAMELS)

NSE	Q	QS_d	DB	DBS_d
> 0.5	91	96	96	96
> 0.6	90	95	95	95
> 0.7	87	93	95	95
> 0.8	76	92	92	94
> 0.9	60	82	86	88

Discussion and conclusion

- Our analysis suggest that long-term catchment fluxes and storage contain sufficient information to reconstruct the flow duration curve to a good level of agreement.
- The following framework is suggested:



- In order to be effective for prediction, our methodology relies on catchment-scale estimates of mean annual fluxes and recession constant. Further investigation of frameworks for their prediction (e.g. Ponce and Shetty model) is needed.
- Alternatively, recently produced data-sets[7] with global-scale values of Q_b, Q_d and k can be used within the proposed framework for the prediction of FDCs at ungauged basins.

Next steps

More work is needed to understand the mechanisms behind these scaling relationships in order to develop a proper statistical framework.

Future work should also extend this analysis to ephemeral streams.

References

1. Yokoo, Yoshiyuki, Murugesu Sivapalan, and Taikan Oki. "Investigating the roles of climate seasonality and landscape characteristics on mean annual and monthly water balances." Journal of Hydrology 357.3-4 (2008): 255-269.
2. Yaeger, Mary, et al. "Exploring the physical controls of regional patterns of flow duration curves–Part 4: A synthesis of empirical analysis, process modeling and catchment classification." Hydrology and Earth System Sciences 16.11 (2012): 4483-4498.
3. Cheng, Lei, et al. "Exploring the physical controls of regional patterns of flow duration curves–Part 1: Insights from statistical analyses." Hydrology and Earth System Sciences 16.11 (2012): 4435-4446.
4. Schaake, John, Shuzheng Cong, and Qingyun Duan. "The US MOPEX data set." *IAHS publication* 307.9 (2006).
5. Addor, Nans, et al. "The CAMELS data set: catchment attributes and meteorology for large-sample studies." *Hydrology and Earth System Sciences (HESS)* 21.10 (2017): 5293-5313.
6. Lyne, V., and M. Hollick. "Stochastic time-variable rainfall-runoff modelling." Institute of Engineers Australia National Conference. Vol. 1979. Barton, Australia: Institute of Engineers Australia, 1979.
7. Beck, Hylke E., et al. "Global patterns in base flow index and recession based on streamflow observations from 3394 catchments." Water Resources Research 49.12 (2013): 7843-7863.

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